

## What I know about lines?

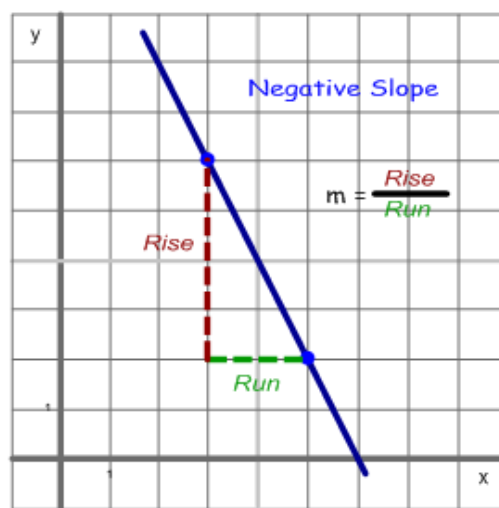
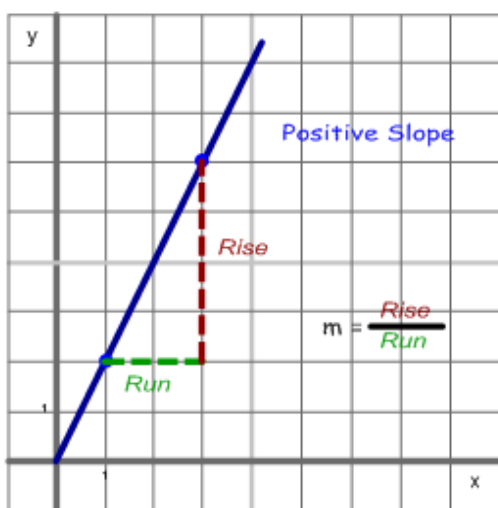
A line can be represented graphically on a coordinate plane or in an algebraic equation.

A line consists of at least two points or coordinates.

A coordinate [or point] is a location on the coordinate plane. It is composed of an x-coordinate and a y-coordinate,  $(x, y)$ .

Label individual points with subscripts, examples:

Point 1  $(x_1, y_1)$    Point 2  $(x_2, y_2)$    Point A  $(x_A, y_A)$    Point B  $(x_B, y_B)$



## **Find the equation of the line between two points using the graphing method.**

1. Plot any two points on a coordinate grid. Label your axis and points.
2. Draw a line connecting the two points.
3. Create a Right Triangle between the two points (draw a horizontal & vertical line at a right angle)
4. Count the distance between the right angle and each point. Label the Rise and Run with values.
5. Find the slope,  $m$ .  $m = \frac{\text{Rise}}{\text{Run}}$  Or compute it with  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .
6. Determine if the line is positive slope or negative slope. Read the graph from right to left.
7. Find where the line crosses the y-axis. This is the y-intercept  $(0, b)$ .
8. Now write the equation  $y = mx + b$  ***Slope – Intercept Form***

## All the Forms of the Equation of the Line:

$$y = m x + b \quad \text{Slope} - \text{Intercept Form}$$

$$A x + B y = C \quad \text{Standard Form}$$

$$y - y_1 = m (x - x_1) \quad \text{Point} - \text{Slope Form}$$

The **y-intercept** is the point [or coordinate]  $(0, y)$  or  $(0, b)$  where a line crosses the y-axis. At this point the x-coordinate is zero and the y-coordinate is b and the unknown value.

The **x-intercept** is the point [or coordinate]  $(x, 0)$  where a line crosses the x-axis. At this point the y-coordinate is zero and the x-coordinate is the unknown value.

**Horizontal lines** cross only the y-axis. Therefore the equation of the line is  $y = b$ , the y-intercept for a horizontal line is  $(0, b)$ .

There **isn't any rise (No Rise)** in a horizontal line therefore

$$\text{the slope is } m = \frac{\text{Rise}}{\text{Run}} = \frac{0}{\text{run}} = 0, \text{ zero.}$$

### Advance Concepts

$$y = 0 \cdot x + b \quad \text{Slope} - \text{Intercept Form} \quad y = b$$

$$0 \cdot x + B y = C \quad \text{Standard Form} \quad y = \frac{C}{B} = b$$

$$y - y_1 = 0 \cdot (x - x_1) \quad \text{Point} - \text{Slope Form} \quad y = y_1 = b$$

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**Vertical lines** cross only the x-axis. Therefore the equation of the line is in the form of  $x = \text{"some value"}$  and the x-intercept is  $(x, 0)$ .

All other lines must cross both axes. Therefore all other lines with have an x-intercept and y-intercept.

There **isn't any run (No Run)** in a vertical line therefore  $m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{rise}}{0} = \text{undefined}$ .

**Find the equation of the line in the Slope-Intercept Form using two points.**

Example 1:

$$\begin{array}{ll} \text{point 1}(-4, 7), & \text{point 2}(3, -2) \\ x_1 = -4, y_1 = 7 & x_2 = 3, y_2 = -2 \end{array}$$

1) Use Slope Formula,  $m = \frac{y_2 - y_1}{x_2 - x_1}$  .  $m = \frac{-2 - 7}{3 - (-4)} = \frac{-9}{7}$  or  $m = -\frac{9}{7}$  .

2) Choose one of the points (3, -2) and substitute the values of (x, y) with the value of slope into the equation of the line:

$$y = m(x) + b \text{ becomes } -2 = \frac{-9}{7}(3) + b \text{ and then solve for } b.$$

$$-2 = \frac{-9}{7}(3) + b \quad \text{a. Simplify}$$

$$-2 = \frac{-27}{7} + b$$

$$-2 = -3 + b \quad \text{b. Add } 3 \text{ to both sides to isolate } b.$$

$$\begin{array}{r} +3 \quad +3 \\ 1 = b \end{array} \text{ rewrite as } b = 1$$

c. Rewrite the equation of the line:  $y = mx + b$  as  $y = \frac{-9}{7}x + 1$

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Transform the slope-intercept form into the standard form of the equation of the line:

$$Ax + By = C$$

$$y = -\frac{9}{7}x + 1$$

$$\begin{array}{r} +9 \\ \frac{7}{7}x \end{array} \quad \begin{array}{r} +9 \\ \frac{7}{7}x \end{array}$$

$$\begin{array}{r} +9 \\ \frac{7}{7}x + y = \end{array} \quad 1$$

$$7 \cdot \left( \frac{9}{7}x + y \right) = 7 \cdot (1)$$

$$9x + 7y = 7 \text{ Standard Form of } y = -\frac{9}{7}x + 1$$

$$Ax + By = C$$

*Standard Form*

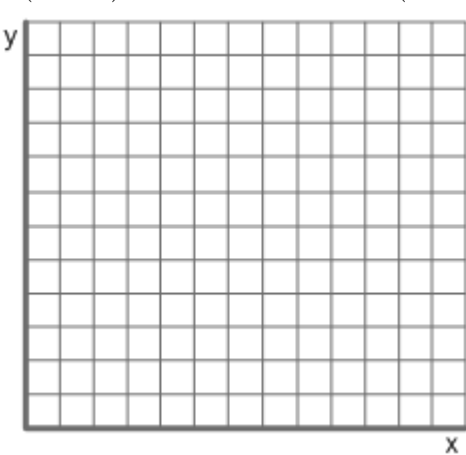
**Finding the x-intercept and y-intercept when the equation is in Standard Form.**

- All line cross the both axes (exception horizontal & vertical lines).
- Find the values of the **x-intercept** and **y-intercept**.

The **y-intercept** is the point [or coordinate]  $(0, y)$  or  $(0, b)$ , where the line crosses the y-axis. The **x-intercept** is the point [or coordinate]  $(x, 0)$ , where the line crosses the x-axis.

### Process

- Create two columns & label y-intercept  $(0, y)$  and x- intercept  $(x, 0)$ .

<b>y-intercept</b> $(0, y)$	$6x + 3y = 36$	<b>x-intercept</b> $(x, 0)$	
$6x + 3y = 36$		$6x + 3y = 36$	
<b>substitute point into equation</b>			
$6 \cdot 0 + 3y = 36$		$6x + 3 \cdot 0 = 36$	
$3y = 36$		$6x = 36$	
$\frac{3y}{3} = \frac{36}{3}$		$\frac{6x}{6} = \frac{36}{6}$	
$y = 12$	$(0, 12)$	$x = 6$	$(6, 0)$
			

**Explain: What I know about lines?**

- The equation of the line is in standard form.
- All line cross the both axes (exception horizontal & vertical lines).
- The y-intercept is  $(0, y)$ .
- The x-intercept is  $(x, 0)$ .

**Process**

- Create two columns & label y-intercept  $(0, y)$  and x- intercept  $(x, 0)$ .
- Write original equation first & then substitute the know value of the appropriate coordinate, y-intercept  $(0, y)$  substitute 0 for x in the equation.
- Simplify equation and solve for y.
- Write the answer as a coordinate with the value of y.  $(0, y)$ .
- Do the same of the x-intercept.
- Plot the coordinate and draw the line.

The Domain is all values of x  
The Range is all values of y.

**Parallel Lines**  $\parallel$  have the same slope and a different y-intercept.

$$\overline{AB} \parallel \overline{CD} \quad m_{AB} = m_{CD}$$

$$y = m_{AB}x + b_{AB} \quad \parallel \quad y = m_{CD}x + b_{CD}$$

**Perpendicular Lines**  $\perp$   $\overline{AB} \perp \overline{CD} \quad m_{CD} = -\frac{1}{m_{AB}} \quad \text{or} \quad m_{AB} = -\frac{1}{m_{CD}}$

$$y = m_{AB}x + b_{AB} \quad \perp \quad y = m_{CD}x + b_{CD}$$

$$y = m_{AB}x + b_{AB} \quad \perp \quad y = -\frac{1}{m_{AB}}x + b_{CD}$$

Examples

Parallel Lines have same slope.	Perpendicular Lines
$\overline{AB} \quad y = -\frac{9}{7}x + 1 \quad m_{AB} = -\frac{9}{7}$	$\overline{AB} \quad y = -\frac{9}{7}x + 1 \quad m_{AB} = -\frac{9}{7}$
$\overline{AB} \parallel \overline{CD} \text{ at } (0, -4)$	$\overline{AB} \perp \overline{CD} \text{ at } (0, 1)$
$\overline{CD} \quad y = -\frac{9}{7}x - 4 \quad m_{CD} = -\frac{9}{7}$	$\overline{CD} \quad y = \frac{7}{9}x + 1 \quad m_{CD} = +\frac{7}{9}$

**Advanced Parallel & Perpendicular Lines at a Particular point,  $(x, y)$  .**

Find a line that is parallel and perpendicular to  $y = -\frac{9}{7}x + 1$  at  $(-7, 5)$

Use the information above:  $m_{AB} = -\frac{9}{7}$        $m_{CD} = +\frac{7}{9}$

Use this the Point-Slope Form of the Equation of the Line.

$$y - y_1 = m(x - x_1) \quad \text{Point - Slope Form}$$

Parallel Line at $(-7, 5)$	Perpendicular Line at $(9, -5)$
$\overline{AB} \quad y = -\frac{9}{7}x + 1 \quad m_{AB} = -\frac{9}{7}$ $y - 5 = -\frac{9}{7}(x - (-7))$ $y - 5 = -\frac{9}{7}(x + 7)$ $y - 5 = -\frac{9}{7}x + (-\frac{9}{7}) \cdot 7$ $y - 5 = -\frac{9}{7}x - 9$ $\quad \quad \quad +5 \quad \quad \quad +5$ $y = -\frac{9}{7}x - 4$ $\overline{AB} \parallel \overline{CD} \text{ at } (-7, 5)$ $\overline{CD} \quad y = -\frac{9}{7}x - 4 \quad m_{CD} = -\frac{9}{7}$ y-intercept is $(0, -4)$	$y - (-5) = \frac{7}{9}(x - 9)$ $y + 5 = \frac{7}{9}(x - 9)$ $y + 5 = \frac{7}{9}x - (\frac{7}{9}) \cdot 9$ $y + 5 = \frac{7}{9}x - 7$ $\quad \quad \quad -5 \quad \quad \quad -5$ $y = \frac{7}{9}x - 12$ $\overline{AB} \perp \overline{CD} \text{ at } (-7, 5)$ $\overline{CD} \quad y = \frac{7}{9}x - 12 \quad m_{CD} = +\frac{7}{9}$ y-intercept is $(0, -12)$